10

Mathematics

Quarter 4 – Module 2
(Week 2)
Calculates a Specified Measure of Position

DepED
DEPARTMENT OF EDUCATION

About the Module

This module was designed and written with you in mind. It is here to help you master the skills in calculating a specified measure of position. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module is divided into two lessons, namely:

Lesson 1 – Measures of Position for Ungrouped Data
Lesson 2 – Measures of Position for Grouped Data

After going through this module, you are expected to:

1. calculate the given measure of position for ungrouped data; and
2. calculate the given measure of position for grouped data.

What I Know (Pre-Test)

Instructions: Read each item carefully. Encircle the letter of the best answer.

1. When the distribution is divided into four equal parts, each score point that describes the distribution is called _____.
   A. Decile B. Percentile C. Quantile D. Quartile
2. It refers to data in the raw or data that you first gather.
   A. Frequency Distribution C. Grouped Data
   B. Organized Data D. Ungrouped Data
3. It is a table which shows that data arranged into different classes and the number of frequencies which fall into each class.
   A. Frequency Distribution C. Grouped Data
   B. Organized Data D. Ungrouped Data

For items 4 to 9, refer to the data below.
Given 50 multiple-choice items in Mathematics test, the scores of 10 students are the following: 35, 38, 48, 21, 18, 28, 46, 22, 43, and 27.

4. What is the position of Q_3?
   A. 5^{th} B. 6^{th} C. 7^{th} D. 8^{th}
5. What is the value of Q_3?
   A. 35 B. 38 C. 43 D. 44
6. What is the position of P_{55}?
   A. 5^{th} B. 6^{th} C. 7^{th} D. 8^{th}
7. Calculate Q_1.
   A. 22 B. 27 C. 28 D. 35
8. Calculate D_4.
   A. 22 B. 27 C. 28 D. 35
9. Calculate P_{79}.
   A. 43 B. 44 C. 46 D. 48

For items 10 to 15, refer to the data below.

Score	Frequency	Lower Boundaries	Cumulative Frequency
46-50	6	45.5	30
41-45	5	40.5	24
36-40	7	35.5	19
31-35	3	30.5	12
26-30	9	25.5	9

10. In solving for the 35^{th} percentile, the lower boundary is ____.
    A. 25.5 B. 30.5 C. 35.5 D. 40.5
11. What class interval should be used in solving for the 1^{st} quartile?
    A. 26-30 B. 31-35 C. 36-40 D. 41-45
12. What cumulative frequency should be used in solving for the 3^{rd} quartile?
    A. 9 B. 12 C. 19 D. 24
13. Calculate the 3^{rd} quartile.
    A. 43 B. 43.5 C. 44 D. 44.5
14. Calculate the 50^{th} percentile.
    A. 34.75 B. 35.5 C. 36.55 D. 37.65
15. Calculate the 6^{th} decile.
    A. 39.8 B. 38.8 C. 37.8 D. 36.8

Lesson 1 Monday	**Measures of Position for Ungrouped Data** **QUARTILE, DECILE AND PERCENTILE**

What I Need to Know

At the end of this lesson, you are expected to:

- calculate the given measure of position for ungrouped data.

What's In

In the previous lesson lessons, you were able to learn quartile, decile and percentile.

Here are some important terms to recall. Answer the following.

_____1. When a distribution is divided into four equal parts, each score point that describes the distribution is called _____.
_____2. These are values that divide a set of data into 100 equal parts.
_____3. It divides the distribution into two equal parts.
_____4. These are values that divide a set of data into 10 equal parts.
_____5. It is the difference between Q_3 and Q_1.

What's New

Think of it!

You are the third tallest student in a group of 10. If you are the 3rd tallest student, therefore 7 students are shorter than you. It means that 70% of the students are shorter than you. if you are the 6th tallest student in a group of 10, how many percent of the students are shorter than you?

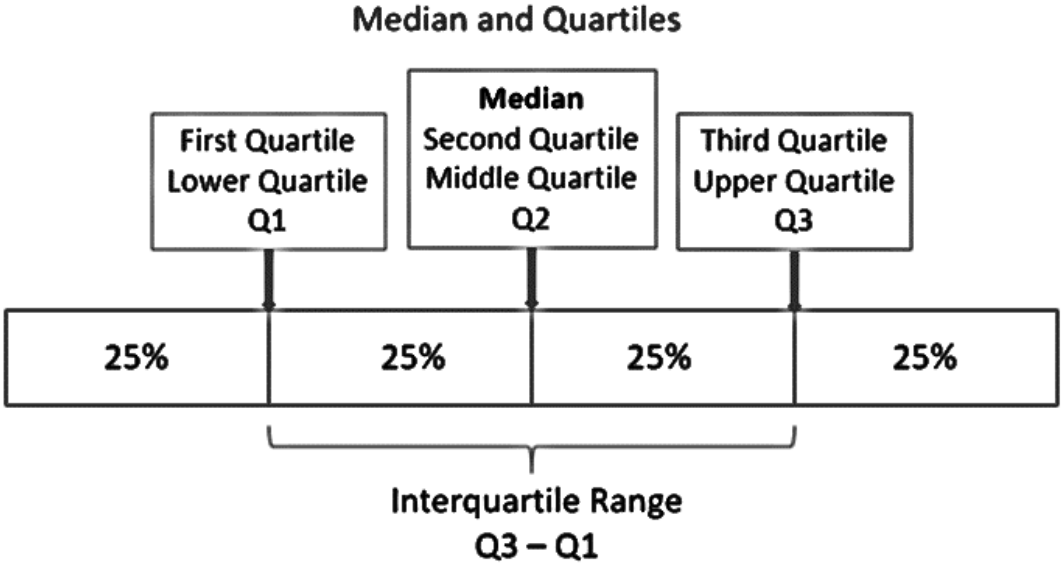
What Is It

When a set of data is arranged from lowest to highest, the distribution can be divided into two, four, ten or a hundred equal parts. The points that are equally divided in the set of data are called *quantiles*. *Quartiles* divide the distribution into four equal parts, *deciles* into ten equal parts, and *percentiles* into one hundred equal parts.

These descriptive measures, quartiles, deciles and percentiles are called fractiles.

QUARTILE FOR UNGROUPED DATA

The quartiles are the score points which divide a distribution into four equal parts.



Mendenhall and Sincich Method

The method of finding the quartile value.

The position of the data in the distribution is obtained by using the following formula, where n is the total number of observations.

Steps in solving quartiles using Mendenhall and Sincich Method.

Step 1. Arrange the data in ascending order.
Step 2. Locate the position of the data in the distribution.
Step 3. Identify the location of the data and get the value.

$$Q_k = \frac{k}{4}(n+1)$$

- If L (lower quartile) falls halfway between two integers, round up, then L^{th} element is the lower quartile value (Q_1).
- If U (upper quartile) falls halfway between two integers, round down, then U^{th} element is the upper quartile value (Q_3).

Illustrative Examples:

Ex. 1. Using Mendenhall and Sincich Method, find the 1st quartile for the following data representing the heights (in centimeters) of 9 high school students.

142 134 162 147 157 137 154 144 149

Solutions:

1. First, arrange the data in ascending order.

134 137 142 144 147 149 154 157 162

2. Locate the position of the data in the distribution.

To locate the position of the data, use

$$\boldsymbol{Q_k = \frac{k}{4}(n+1)}$$

Since we need to find the 1st quartile and we have 9 given data, k = 1 and n = 9.

$$\boldsymbol{Q_1 = \frac{1}{4}(9+1)}$$

$$\boldsymbol{Q_1 = \frac{1}{4}(10)}$$

$$\boldsymbol{Q_1 = \frac{10}{4}}$$

$$\boldsymbol{Q_1 = 2.5 \approx 3}$$

If L (lower quartile) falls halfway between two integers, round up, then L^{th} element is the lower quartile value (Q_1). The computed value 2.5 becomes 3 after rounding up.

3. Identify the location of the data and get the value.

134 137 142 144 147 149 154 157 162

The lower quartile value (Q_1) is the 3rd data element, thus Q_1= 142.

Ex. 2. Using Mendenhall and Sincich Method, find the 3rd quartile for the following data representing the heights (in centimeters) of 10 high school students.

142 134 162 147 157 137 154 144 149 150

Solutions:

1. First, arrange the data in ascending order.

134 137 142 144 147 149 150 154 157 162

2. Locate the position of the data in the distribution, use:

$$\boldsymbol{Q_k = \frac{k}{4}(n+1)}$$

Since we need to find the 1st quartile and we have 9 given data, k = 3 and n = 10.

$$\boldsymbol{Q_3 = \frac{3}{4}(10+1)}$$

$$\boldsymbol{Q_3 = \frac{3}{4}(11)}$$

$$\boldsymbol{Q_3 = \frac{33}{4}}$$

$$\boldsymbol{Q_3 = 8.25 \approx 8}$$

If U (upper quartile) falls halfway between two integers, round down, then U^{th} element is the upper quartile value (Q_3). The computed value 8.25 becomes 8 after rounding down.

3. Identify the location of the data and get the value.

134 137 142 144 147 149 150 154 157 162

The upper quartile value (Q_3) is the 8th data element, thus Q_3= 154.

Linear Interpolation
A method in finding the quartile value when the result of Mendenhall is decimal.

Step 1. Arrange the data in ascending order.
Step 2. Locate the position of the data in the distribution.
Step 3. If the result is decimal, proceed for linear interpolation.

Steps in solving quartiles using Linear Interpolation.
1. Find the difference between the two values wherein Q_k is situated.
2. Multiply the result in Step 1 by the decimal part obtained in step 2.
3. Add the result in step 2 to the smaller number in step 1.

Illustrative Examples:

Ex. 1. Using Linear Interpolation, find the 1st quartile for the following data representing the heights (in centimeters) of 9 high school students.

142 134 162 147 157 137 154 144 149

Solutions:

Step 1. First, arrange the data in ascending order.

134 137 142 144 147 149 154 157 162

Step 2. Locate the position of the data in the distribution, use:

$$\boldsymbol{Q_k = \frac{k}{4}(n+1)}$$

Since we need to find the 1st quartile and we have 9 given data, k = 1 and n = 9.

$$\boldsymbol{Q_1 = \frac{1}{4}(9+1)}$$

$$\boldsymbol{Q_1 = \frac{1}{4}(10)}$$

$$\boldsymbol{Q_1 = \frac{10}{4}}$$

$$\boldsymbol{Q_1 = 2.5}$$

Step 3. If the result is decimal, proceed for linear interpolation.

134 137 142 144 147 149 154 157 162

Steps in solving quartiles using Linear Interpolation.
1. Find the difference between the two values wherein Q_k is situated.
Q_1 is between 137 and 142, therefore;

= 142 – 137
= 5

2. Multiply the result in Step 1 by the decimal part obtained in step 2.

= 5 (0.5)
= 2.5

3. Add the result in step 2 to the smaller number in step 1.

= 2.5 + 137
= 139.5

Therefore, the value of Q_1 is 139.5.

Ex. 2. Using Linear Interpolation, find the 3rd quartile for the following data representing the heights (in centimeters) of 10 high school students.

142 134 162 147 157 137 154 144 149 150

Solutions:

Step 1. First, arrange the data in ascending order.

134 137 142 144 147 149 150 154 157 162

Step 2. Locate the position of the data in the distribution, use:

$$\boldsymbol{Q_k = \frac{k}{4}(n+1)}$$

Since we need to find the 1st quartile and we have 9 given data, k = 3 and n = 10.

$$\boldsymbol{Q_3 = \frac{3}{4}(10+1)}$$

$$\boldsymbol{Q_3 = \frac{3}{4}(11)}$$

$$\boldsymbol{Q_3 = \frac{33}{4}}$$

$$\boldsymbol{Q_3 = 8.25}$$

Step 3. If the result is decimal, proceed for linear interpolation.

134 137 142 144 147 149 150 154 157 162

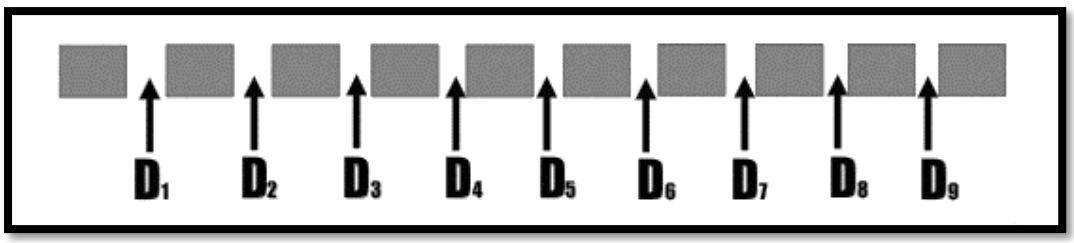
Steps in solving quartiles using Linear Interpolation.

1. Find the difference between the two values wherein Q_k is situated.
Q_3 is between 154 and 157, therefore;
= 157 – 154
= 3
2. Multiply the result in Step 1 by the decimal part obtained in step 2.
= 3 (0.25)
= 0.75
3. Add the result in step 2 to the smaller number in step 1.
= 0.75 + 154
= 154.75

Therefore, the value of Q_3 is 154.75.

DECILE FOR UNGROUPED DATA

The decile are the nine score points which divides a distribution into ten equal parts.



To calculate the position of deciles, use the formula;

$$\boldsymbol{D_k = \frac{k}{10}(n+1)}$$

where k is the position of the decile and n is the total number of data.

Steps in solving deciles:

Step 1. Arrange the data in ascending order.
Step 2. Locate the position of the data in the distribution. If the answer is decimal, round off to the nearest integer.
Step 3. Identify the location of the data and get the value.

Illustrative Examples:

Ex. 1. Find the 3rd decile of the following test scores of a random sample of 15 students: 22, 31, 24, 28, 47, 34, 40, 39, 30, 29, 17, 23, 32, 25 and 45.

Solutions:

Step 1. Arrange the data in ascending order.

17 22 23 24 25 28 29 30 31 32 34 39 40 45 47

Step 2. Locate the position of the data in the distribution. If the answer is decimal, round off to the nearest integer.

Since we need to solve for the 3rd decile and there are 10 test scores, $k = 3$ and $n = 15$.

$$\boldsymbol{D_k = \frac{k}{10}(n+1)}$$

$$D_3 = \frac{3}{10}(15+1)$$

$$D_3 = \frac{3}{10}(16)$$

$$D_3 = \frac{48}{10}$$

$$D_3 = 4.8 \approx 5$$

Step 3. Identify the location of the data and get the value.

17 22 23 24 25 28 29 30 31 32 34 39 40 45 47

D_3 is the 5th data element, therefore, $D_3 = 25$.

Ex. 2. Find the 6th decile of the following test scores of a random sample of 11 students: 34, 40, 39, 42, 30, 29, 17, 23, 32, 25 and 45.

Solutions:

Step 1. Arrange the data in ascending order.

17 23 25 29 30 32 34 39 40 42 45

Step 2. Locate the position of the data in the distribution. If the answer is decimal, round off to the nearest integer.

Since we need to solve for the 6th decile and there are 11 test scores, $k = 6$ and $n = 11$.

$$\boldsymbol{D_k = \frac{k}{10}(n+1)}$$

$$D_6 = \frac{6}{10}(11+1)$$

$$D_6 = \frac{6}{10}(12)$$

$$D_6 = \frac{72}{10}$$

$$D_6 = 7.2 \approx 7$$

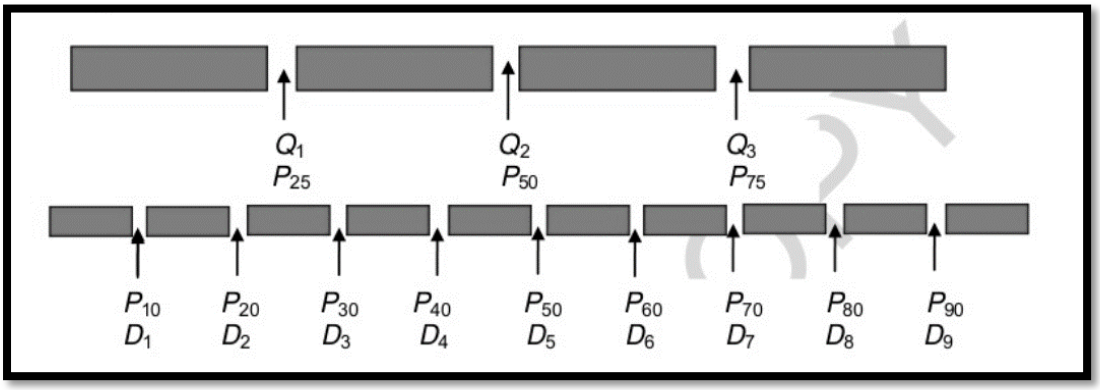
Step 3. Identify the location of the data and get the value.

17 23 25 29 30 32 34 39 40 42 45

D_6 is the 7th data element, therefore, $D_6 = 34$.

PERCENTILE FOR UNGROUPED DATA

The percentile are the ninety-nine score points which divides a distribution into one hundred equal parts.



To calculate the position of percentiles, use the formula;

$$\boldsymbol{P_k = \frac{k}{100}(n+1)}$$

where k is the position of the decile and n is the total number of data.

Steps in solving deciles:

Step 1. Arrange the data in ascending order.
Step 2. Locate the position of the data in the distribution. If the answer is decimal, round off to the nearest integer.
Step 3. Identify the location of the data and get the value.

Illustrative Examples:

Ex. 1. Find the 40th percentile of the following test scores of a random sample of 20 students: 11, 41, 23, 17, 30, 14, 10, 26, 38, 20, 34, 40, 39, 30, 29, 17, 23, 32, 25 and 45.

Solutions:

Step 1. Arrange the data in ascending order.

10 11 14 17 17 20 23 23 25 26 29 30 30 32 34 38 39 40 41 45

Step 2. Locate the position of the data in the distribution. If the answer is decimal, round off to the nearest integer.

Since we need to solve for the 40th percentile and there are 10 test scores, $k = 40$ and $n = 20$.

$$\boldsymbol{P_k = \frac{k}{100}(n+1)}$$

$$P_{40} = \frac{40}{100}(20+1)$$

$$P_{40} = \frac{40}{100}(21)$$

$$P_{40} = \frac{840}{100}$$

$$P_{40} = 8.4 \approx 8$$

Step 3. Identify the location of the data and get the value.

10 11 14 17 17 20 23 23 25 26 29 30 30 32 34 38 39 40 41 45

P_{40} is the 8th data element, therefore, $P_{40} = 23$.

Ex. 2. Find the 60th percentile of the following test scores of a random sample of nine students: 40, 39, 30, 29, 17, 23, 32, 25 and 45.

Solutions:

Step 1. Arrange the data in ascending order.

17 23 25 29 30 32 39 40 45

Step 2. Locate the position of the data in the distribution. If the answer is decimal, round off to the nearest integer.

Since we need to solve for the 40th percentile and there are 9 test scores, $k = 60$ and $n = 9$.

$$\boldsymbol{P_k = \frac{k}{100}(n+1)}$$

$$P_{60} = \frac{60}{100}(9+1)$$

$$P_{40} = \frac{60}{100}(10)$$

$$P_{40} = \frac{600}{100}$$

$$P_{60} = 6$$

Step 3. Identify the location of the data and get the value.

17 23 25 29 30 32 39 40 45

P_{60} is the 6th data element, therefore, $P_{60} = 32$.

What's More

Activity 1. NOW IT'S YOUR TURN!

A. The reaction time (in seconds) of 20 jeepney drivers for the green light are as follows:

1, 5, 5, 4, 8,10, 10, 3, 3, 2, 2, 5, 4, 8, 4, 7, 4, 3, 5, 5

Calculate the following:

1. D_4
2. D_7
3. P_{10}
4. P_{75}

B. Use linear interpolation in solving the 1st and 3rd quartile of the following test scores of a random 9 students:

18 20 23 28 29 30 33 35 43

What I Need to Remember

Quartile for Ungrouped Data

$$Q_k = \frac{k}{4}(n+1)$$

Decile for Ungrouped Data

$$D_k = \frac{k}{10}(n+1)$$

Percentile for Ungrouped Data

$$P_k = \frac{k}{100}(n+1)$$

Lesson 2 Tuesday	**Measures of Position for Grouped Data**
	QUARTILE, DECILE AND PERCENTILE

What I Need to Know

At the end of this lesson, you are expected to:

- calculate the given measure of position for grouped data.

What's In

Do you have a sharp memory?

The following are the number of years of service of 20 school teachers:

4	5	6	6	7	8	10	10	11	16	17	17	18	19	20	20	21	23	25	30

Find: 1. Q_3 2. D_8 3. P_{70}

What's New

Before we calculate the given measure of position for grouped data, let us try to answer this question.

What is the difference between grouped and ungrouped data?

What Is It

Let us try to answer the previous activity.

Ungrouped data are data in the raw or data that you first gather.
Grouped data are data that has been organized into groups known as classes.

FREQUENCY DISTRIBUTION

In calculating measure of position for grouped data, we will be using frequency distribution table. A frequency distribution table is a table which shows that data arranged into different classes and the number of frequencies which fall into each class.

Quartile of Grouped Data

Recall that quartiles divide the distribution into four equal parts.
The following is used in calculating quartiles of grouped data:

$$\boldsymbol{Q_k = LB + \left[\frac{\frac{kN}{4} - cf_b}{f_{Q_k}}\right] i}$$

where: LB = lower boundary of Q_k class
N = total frequency
cf_b = cumulative frequency of the class of the class before the Q_k class
f_{Q_k} = frequency of the Q_k class
i = size of the class interval
k = nth quartile, where n=1, 2 and 3

Formula to calculate the Q_k class

$$\boldsymbol{Q_k\ Class = \frac{kN}{4}}$$

where: N = total frequency
k = nth quartile, where n=1, 2 and 3

Decile of Grouped Data

Deciles are values that divide the distribution into 10 equal parts.
The following is used in calculating decile of grouped data:

$$\boldsymbol{D_k = LB + \left[\frac{\frac{kN}{10} - cf_b}{f_{D_k}}\right] i}$$

where: LB = lower boundary of D_k class
N = total frequency
cf_b = cumulative frequency of the class of the class before the D_k class
f_{D_k} = frequency of the D_k class
i = size of the class interval
k = nth quartile, where n=1, 2, 3, 4, 5, 6, 7, 8 and 9

Formula to calculate the $D_k$class

$$\boldsymbol{D_k\ Class = \frac{kN}{10}}$$

where: N = total frequency
k = nth decile, where n=1, 2, 3, 4, 5, 6, 7, 8 and 9

Percentile of Grouped Data

Percentiles are values that divide the distribution into 100 equal parts.
The following is used in calculating percentile of grouped data:

$$P_k = LB + \left[\frac{\frac{kN}{100} - cf_b}{f_{P_k}}\right] i$$

where: LB = lower boundary of P_k class
N = total frequency
cf_b = cumulative frequency of the class of the class before the P_k class
f_{P_k} = frequency of the P_k class
i = size of the class interval
k = nth quartile, where n=1, 2, …, 98 and 99

Formula to calculate the P_k class

$$P_k\ Class = \frac{kN}{10}$$

where: N = total frequency
k = nth percentile, where n=1, 2, …, 98 and 99

Steps in calculating a specified measure of position for grouped data.

Step 1: Determine the lower boundaries;
Step 2: Determine the cumulative frequency;
Step 3: Calculate the given class;
Step 4: Locate the class interval where the given class is situated; and
Step 5: Calculate the value using the formula for grouped data.

Illustrative Examples

Ex. 1. Calculate the Q_3 of the Mathematics test scores of 50 students.

Scores	Frequency
46-50	5
41-45	6
36-40	9
31-35	10
26-30	13
21-25	7

Solutions:

Step 1: Determine the lower boundaries;

To determine the lower boundary (LB), subtract 0.5 to the smallest number per class interval.

Scores	Frequency	Lower Boundaries (LB)	
46-50	5	45.5	← 46 – 0.5 = 45.5
41-45	6	40.5	← 41 – 0.5 = 40.5
36-40	9	35.5	← 36 – 0.5 = 35.5
31-35	10	30.5	← 31 – 0.5 = 30.5
26-30	13	25.5	← 26 – 0.5 = 25.5
21-25	7	20.5	← 21 – 0.5 = 20.5

Step 2: Determine the cumulative frequency;

Scores	Frequency	Lower Boundaries (LB)	Cumulative Frequency	
46-50	5	45.5	50	Add 45 and 5
41-45	6	40.5	45	Add 39 and 6
36-40	9	35.5	39	Add 30 and 9
31-35	10	30.5	30	Add 20 and 10
26-30	13	25.5	20	Add 7 and 13
21-25	7	20.5	7	Copied from the frequency

Step 3: Calculate the given class;

Since we are calculating Q_3, we will use the formula for quartile.

Given: $k = 3;\ N = 50$

$$\boldsymbol{Q_k\ Class = \frac{kN}{4}}$$

$$Q_3\ Class = \frac{3(50)}{4}$$

$$Q_3\ Class = \frac{150}{4}$$

$$Q_3\ Class = 37.5$$

This means that we need to find the class interval where 37.5th score is obtained.

Step 4: Locate the class interval where the given class is situated; and

Scores	Frequency	Lower Boundaries (LB)	Cumulative Frequency	
46-50	5	45.5	50	
41-45	6	40.5	45	
36-40	9	35.5	39	(31st-39th score)
31-35	10	30.5	30	(21st-30th score)
26-30	13	25.5	20	(8th-20th score)
21-25	7	20.5	7	(1st-7th score)

Use the cumulative frequency in finding the 37.5th score. Since 37.5th score is obtained between 31st and 39th score, we will use the class interval 36-40.

Step 5: Calculate the value using the formula for grouped data.

Since we are calculating the value of Q_3, we will use the formula for quartile for grouped data.

$$\boldsymbol{Q_k = LB + \left[\frac{\frac{kN}{4} - cf_b}{f_{Q_k}}\right] i}$$

where: LB = lower boundary of Q_k class
N = total frequency
cf_b = cumulative frequency of the class of the class before the Q_k class
f_{Q_k} = frequency of the Q_k class
i = size of the class interval
k = nth quartile, where n=1, 2 and 3

Scores	Frequency	Lower Boundaries (LB)	Cumulative Frequency	
46-50	5	45.5	50	
41-45	6	40.5	45	
36-40	9	35.5	39	(31st-39th score)
31-35	10	30.5	30	(21st-30th score)
26-30	13	25.5	20	(8th-20th score)
21-25	7	20.5	7	(1st-7th score)

Given: $LB = 35.5, k = 3, N = 50, cf_b = 30, i = 5, f_{Q_3} = 9$

$$Q_3 = LB + \left[\frac{\frac{kN}{4} - cf_b}{f_{Q_3}}\right] i$$

$$Q_3 = 35.5 + \left[\frac{\frac{3(50)}{4} - 30}{9}\right] 5$$

$$Q_3 = 35.5 + \left[\frac{\frac{150}{4} - 30}{9}\right] 5$$

$$Q_3 = 35.5 + \left[\frac{37.5 - 30}{9}\right] 5$$

$$Q_3 = 35.5 + \left[\frac{7.5}{9}\right] 5$$

$$Q_3 = 35.5 + [0.83]5$$

$$Q_3 = 35.5 + 4.15$$

$$\boldsymbol{Q_3 = 39.65}$$

Therefore, the value of Q_3 is 39.65.

Ex. 2. Calculate the D_4 of the number of employees in 50 companies belonging to a certain industry.

Number of Employees	Frequency
41-45	10
36-40	4
31-35	8
26-30	12
21-25	11
16-20	5

Solutions:

Step 1: Determine the lower boundaries;

To determine the lower boundary (LB), subtract 0.5 to the smallest number per class interval.

Scores	Frequency	Lower Boundaries (LB)	
41-45	10	40.5	41 – 0.5 = 40.5
36-40	4	35.5	36 – 0.5 = 35.5
31-35	8	30.5	31 – 0.5 = 30.5
26-30	12	25.5	26 – 0.5 = 25.5
21-25	11	20.5	21 – 0.5 = 20.5
16-20	5	15.5	16 – 0.5 = 15.5

Step 2: Determine the cumulative frequency;

Scores	Frequency	Lower Boundaries (LB)	Cumulative Frequency	
41-45	10	40.5	50	Add 40 and 10
36-40	4	35.5	40	Add 36 and 4
31-35	8	30.5	36	Add 28 and 8
26-30	12	25.5	28	Add 16 and 12
21-25	11	20.5	16	Add 5 and 11
16-20	5	15.5	5	Copied from the frequency

Step 3: Calculate the given class;

Since we are calculating D_4, we will use the formula for decile.

Given: $k = 4;\ N = 50$

$$\boldsymbol{D_k\ Class = \frac{kN}{10}}$$

$$D_4\ Class = \frac{4(50)}{10}$$

$$D_4\ Class = \frac{200}{10}$$

$$D_4\ Class = 20$$

This means that we need to find the class interval where 20th score is obtained.

Step 4: Locate the class interval where the given class is situated; and

Scores	Frequency	Lower Boundaries (LB)	Cumulative Frequency	
41-45	10	40.5	50	
36-40	4	35.5	40	
31-35	8	30.5	36	(29st-36th score)
26-30	12	25.5	28	(17th-28th score)
21-25	11	20.5	16	(6th-16th score)
16-20	5	15.5	5	(1st-5th score)

Use the cumulative frequency in finding the 20th score. Since the 20th score is obtained between 17th and 28th score, we will use the class interval 26-30.

Step 5: Calculate the value using the formula for grouped data.

Since we are calculating the value of D_4, we will use the formula for decile for grouped data.

$$\boldsymbol{D_k = LB + \left[\frac{\frac{kN}{10} - cf_b}{f_{D_k}}\right] i}$$

where: LB = lower boundary of D_k class
N = total frequency
cf_b = cumulative frequency of the class of the class before the D_k class
f_{D_k} = frequency of the D_k class
i = size of the class interval
k = nth decile, where n=1, 2, 3, 4, 5, 6, 7, 8 and 9

Scores	**Frequency**	**Lower Boundaries (LB)**	**Cumulative Frequency**	
41-45	10	40.5	50	
36-40	4	35.5	40	
31-35	8	30.5	36	(29st-36th score)
26-30	12	25.5	28	(17th-28th score)
21-25	11	20.5	16	(6th-16th score)
16-20	5	15.5	5	(1st-5th score)

Given: $LB = 25.5, k = 4, N = 50, cf_b = 16, i = 5, f_{D_4} = 12$

$$D_4 = LB + \left[\frac{\frac{kN}{10} - cf_b}{f_{D_4}}\right] i$$

$$D_4 = 25.5 + \left[\frac{\frac{4(50)}{10} - 16}{12}\right] 5$$

$$D_4 = 25.5 + \left[\frac{\frac{200}{10} - 16}{12}\right] 5$$

$$D_4 = 25.5 + \left[\frac{20 - 16}{12}\right] 5$$

$$D_4 = 25.5 + \left[\frac{4}{12}\right] 5$$

$$D_4 = 25.5 + [0.33]5$$

$$D_4 = 25.5 + 1.65$$

$$\boldsymbol{D_4 = 27.15}$$

Therefore, the value of D_3 is 27.15.

Ex. 3. Calculate the P_{80} of the Mathematics test scores of 50 students.

Scores	Frequency
46-50	4
41-45	8
36-40	11
31-35	9
26-30	12
21-25	6

Solutions:

Step 1: Determine the lower boundaries;

To determine the lower boundary (LB), subtract 0.5 to the smallest number per class interval.

Scores	Frequency	Lower Boundaries (LB)	
46-50	4	45.5	46 – 0.5 = 45.5
41-45	8	40.5	41 – 0.5 = 40.5
36-40	11	35.5	36 – 0.5 = 35.5
31-35	9	30.5	31 – 0.5 = 30.5
26-30	12	25.5	26 – 0.5 = 25.5
21-25	6	20.5	21 – 0.5 = 20.5

Step 2: Determine the cumulative frequency;

Scores	Frequency	Lower Boundaries (LB)	Cumulative Frequency	
46-50	4	45.5	50	Add 46 and 4
41-45	8	40.5	46	Add 38 and 8
36-40	11	35.5	38	Add 27 and 11
31-35	9	30.5	27	Add 18 and 9
26-30	12	25.5	18	Add 6 and 12
21-25	6	20.5	6	Copied from the frequency

Step 3: Calculate the given class;

Since we are calculating P_{80}, we will use the formula for percentile.

Given: $k = 80$; $N = 50$

$$\boldsymbol{P_k\ Class} = \frac{\boldsymbol{kN}}{\boldsymbol{100}}$$

$$P_{80}\ Class = \frac{80(50)}{100}$$

$$P_{80}\ Class = \frac{4000}{100}$$

$$P_{80}\ Class = 40$$

This means that we need to find the class interval where 40th score is obtained.

Step 4: Locate the class interval where the given class is situated; and

Scores	Frequency	Lower Boundaries (LB)	Cumulative Frequency	
46-50	4	45.5	50	
41-45	8	40.5	46	(39th-46th score)
36-40	11	35.5	38	(28th-38th score)
31-35	9	30.5	27	(19th-27th score)
26-30	12	25.5	18	(7th-18th score)
21-25	6	20.5	6	(1st-6th score)

Use the cumulative frequency in finding the 40th score.
Since 40th score is obtained between 39th and 46th score, we will use the class interval 41-45.

Step 5: Calculate the value using the formula for grouped data.

Since we are calculating the value of P_{80}, we will use the formula for percentile for grouped data.

$$\boldsymbol{P_k = LB + \left[\frac{\frac{kN}{100} - cf_b}{f_{P_k}}\right] i}$$

where: LB = lower boundary of P_k class
N = total frequency
cf_b = cumulative frequency of the class of the class before the P_k class
f_{P_k} = frequency of the P_k class
i = size of the class interval
k = nth percentile, where n=1, 2, …, 98 and 99

Scores	Frequency	Lower Boundaries (LB)	Cumulative Frequency	
46-50	4	45.5	50	
41-45	8	40.5	46	(39th-46th score)
36-40	11	35.5	38	(28th-38th score)
31-35	9	30.5	27	(19th-27th score)
26-30	12	25.5	18	(7th-18th score)
21-25	6	20.5	6	(1st-6th score)

Given: $LB = 40.5, k = 80, N = 50, cf_b = 38, i = 5, f_{Q_3} = 8$

$$P_{80} = LB + \left[\frac{\frac{kN}{100} - cf_b}{f_{P_{80}}}\right] i$$

$$P_{80} = 40.5 + \left[\frac{\frac{80(50)}{100} - 38}{8}\right] 5$$

$$P_{80} = 40.5 + \left[\frac{\frac{4000}{100} - 38}{8}\right] 5$$

$$P_{80} = 40.5 + \left[\frac{40-38}{8}\right]5$$

$$P_{80} = 40.5 + \left[\frac{2}{8}\right]5$$

$$P_{80} = 40.5 + [0.25]5$$

$$P_{80} = 40.5 + 1.25$$

$$\boldsymbol{P_{80} = 41.75}$$

Therefore, the value of P_{80} is 41.75.

What's More

Activity 2. NOW IT'S YOUR TURN!

Complete the table below and find the quartiles, deciles and percentiles indicated below based on the given distribution table.

Grades	Frequency	Lower Boundaries	Cumulative Frequency
96-100	8		
91-95	7		
86-90	12		
81-85	9		
76-80	14		
71-75	16		
66-70	11		
61-65	8		
56-60	6		
51-55	9		

1. Q_1
2. D_4
3. P_{25}
4. P_{92}

What I Need to Remember

Quartile of Grouped Data

$$Q_k = LB + \left[\frac{\frac{kN}{4} - cf_b}{f_{Q_k}}\right]i$$

Decile of Grouped Data

$$D_k = LB + \left[\frac{\frac{kN}{10} - cf_b}{f_{D_k}}\right]i$$

Percentile of Grouped Data

$$P_k = LB + \left[\frac{\frac{kN}{100} - cf_b}{f_{P_k}}\right] i$$

Friday	**Measures of Position for Grouped and Ungrouped Data**

What I can do

Activity 3

The weights of the students in a certain class are the following:
69, 70, 72, 65, 80, 87, 66, 61, 68, 63, 71,75, 50, 57, 56

Calculate the following.

1. 1st quartile
2. 7th decile
3. 45th percentile

Activity 4

Complete the table below and find the 3rd quartile, 6th decile and 72nd percentile for the following distribution.

Class Interval	Frequency	Lower Boundaries	Cumulative Frequency
96-100	8		
91-95	7		
86-90	12		
81-85	9		
76-80	14		
71-75	16		
66-70	11		
61-65	8		
56-60	6		
51-55	9		

Assessment (Post Test)

Instructions: Read each item carefully. Choose the letter of the correct answer and write it on a separate answer sheet.

1. When the distribution is divided into hundred equal parts, each score point that describes the distribution is called _____.
   A. Decile B. Percentile C. Quantile D. Quartile
2. It refers to data that has been organized into groups known as classes.
   A. Frequency Distribution C. Grouped Data
   B. Organized Data D. Ungrouped Data
3. It is a table which shows that data arranged into different classes and the number of frequencies which fall into each class.
   A. Frequency Distribution C. Grouped Data
   B. Organized Data D. Ungrouped Data

For items 4 to 9, refer to the data below.
Given 50 multiple-choice items in Mathematics test, the scores of 15 students are the following: 44, 35, 23, 38, 48, 21, 18, 16, 28, 46, 22, 43, 21, 27, 22.

4. What is the position of Q_1?
   A. 2^{nd} B. 3^{rd} C. 4^{th} D. 5^{th}
5. What is the value of Q_1?
   A. 16 B. 18 C. 21 D. 22
6. What is the value of Q_3?
   A. 28 B. 35 C. 38 D. 43
7. What is the position of D_6?
   A. 11^{th} B. 10^{th} C. 9^{th} D. 8^{th}
8. Calculate D_7.
   A. 28 B. 35 C. 38 D. 43
9. Calculate P_{83}.
   A. 43 B. 44 C. 46 D. 48

For items 10 to 15, refer to the data below.

Score	Frequency	Lower Boundaries	Cumulative Frequency
46-50	6	45.5	30
41-45	5	40.5	24
36-40	7	35.5	19
31-35	3	30.5	12
26-30	9	25.5	9

10. In solving for the 60^{th} percentile, the lower boundary is ____.
    B. 25.5 B. 30.5 C. 35.5 D. 40.5
11. What class interval should be used in solving for the 3^{rd} quartile?
    A. 26-30 B. 31-35 C. 36-40 D. 41-45
12. What cumulative frequency should be used in solving for the 40^{th} percentile?
    B. 9 B. 12 C. 19 D. 24
13. Calculate the 3^{rd} quartile.
    A. 43 B. 43.5 C. 44 D. 44.5
14. Calculate the 8^{th} decile.
    B. 44.5 B. 45.5 C. 46.5 D. 47.5
15. Calculate the 60^{th} percentile.
    A. 39.8 B. 38.8 C. 37.8 D. 36.8

Answer Key

Remember: This portion of the module contains all the answers. Your **HONESTY** is required.

Activity 1.A.

1. D_4

$$D_4 = \frac{4}{10}(20+1)$$
$$D_4 = \frac{4}{10}(21)$$
$$D_4 = \frac{84}{10}$$
$$D_4 = 8.4 \approx 8$$

D_4 is the 8th element, thus, $D_4 = 4$.

2. D_7

$$D_7 = \frac{7}{10}(20+1)$$
$$D_7 = \frac{7}{10}(21)$$
$$D_7 = \frac{147}{10}$$
$$D_7 = 14.7 \approx 15$$

D_7 is the 15th element, thus, $D_7 = 5$.

3. P_{10}

$$P_{10} = \frac{10}{100}(20+1)$$
$$P_{10} = \frac{10}{100}(21)$$
$$P_{10} = \frac{210}{100}$$
$$P_{10} = 2.1 \approx 2$$

P_{10} is the 2nd element, thus, $P_{10} = 2$.

4. P_{75}

$$P_{75} = \frac{75}{100}(20+1)$$
$$P_{75} = \frac{75}{100}(21)$$
$$P_{75} = \frac{1575}{100}$$
$$P_{75} = 15.75 \approx 16$$

P_{75} is the 16th element, thus, $P_{75} = 7$.

Activity 1.B.

1. Q_1

$$Q_1 = \frac{1}{4}(9+1)$$
$$Q_1 = \frac{1}{4}(10)$$
$$Q_1 = \frac{10}{4}$$
$$Q_1 = 2.5$$

Interpolate:

Step 1: $23 - 20 = 3$
Step 2: $3(0.5) = 1.5$
Step 3: $1.5 + 20 = 21.5$

Therefore, $Q_1 = 21.5$.

2. Q_3

$$Q_3 = \frac{3}{4}(9+1)$$
$$Q_3 = \frac{3}{4}(10)$$
$$Q_3 = \frac{30}{4}$$
$$Q_3 = 7.5$$

Interpolate:

Step 1: $35 - 33 = 2$
Step 2: $2(0.5) = 1$
Step 3: $1 + 33 = 34$

Therefore, $Q_3 = 34$.

Activity 2.

Grades	Frequency	Lower Boundaries	Cumulative Frequency
96-100	8	95.5	100
91-95	7	90.5	92
86-90	12	85.5	85
81-85	9	80.5	73
76-80	14	75.5	64
71-75	16	70.5	50
66-70	11	65.5	34
61-65	8	60.5	23
56-60	6	55.5	15
51-55	9	50.5	9

Activity 2

1. Q_1

$$Q_1 = LB + \left[\frac{\frac{kN}{4} - cf_b}{f_{Q_1}}\right] i$$

$$Q_1 = 65.5 + \left[\frac{\frac{1(100)}{4} - 23}{11}\right] 5$$

$$Q_1 = 65.5 + \left[\frac{\frac{100}{4} - 23}{11}\right] 5$$

$$Q_1 = 65.5 + \left[\frac{25 - 23}{11}\right] 5$$

$$Q_1 = 65.5 + \left[\frac{2}{11}\right] 5$$

$$Q_1 = 65.5 + [0.18]5$$

$$Q_1 = 65.5 + 0.9$$

$$\boldsymbol{Q_1 = 66.4}$$

2. D_4

$$D_4 = LB + \left[\frac{\frac{kN}{10} - cf_b}{f_{D_4}}\right] i$$

$$D_4 = 70.5 + \left[\frac{\frac{4(100)}{10} - 34}{16}\right] 5$$

$$D_4 = 70.5 + \left[\frac{\frac{400}{10} - 34}{16}\right] 5$$

$$D_4 = 70.5 + \left[\frac{40 - 34}{16}\right] 5$$

$$D_4 = 70.5 + \left[\frac{6}{16}\right] 5$$

$$D_4 = 70.5 + [0.38]5$$

$$D_4 = 70.5 + 1.9$$

$$\boldsymbol{D_4 = 72.4}$$

3. P_{25}

$$P_{25} = LB + \left[\frac{\frac{kN}{100} - cf_b}{f_{P_{25}}}\right] i$$

$$P_{25} = 65.5 + \left[\frac{\frac{25(100)}{100} - 23}{11}\right] 5$$

$$P_{25} = 65.5 + \left[\frac{\frac{2500}{100} - 23}{11}\right] 5$$

$$P_{25} = 65.5 + \left[\frac{25 - 23}{11}\right] 5$$

$$P_{25} = 65.5 + \left[\frac{2}{11}\right] 5$$

$$P_{25} = 65.5 + [0.18]5$$

$$P_{25} = 65.5 + 0.9$$

$$\boldsymbol{P_{25} = 66.4}$$

4. P_{92}

$$P_{92} = LB + \left[\frac{\frac{kN}{100} - cf_b}{f_{P_{92}}}\right] i$$

$$P_{92} = 90.5 + \left[\frac{\frac{92(100)}{100} - 85}{7}\right] 5$$

$$P_{92} = 90.5 + \left[\frac{\frac{9200}{100} - 85}{7}\right] 5$$

$$P_{92} = 90.5 + \left[\frac{92 - 85}{7}\right] 5$$

$$P_{92} = 90.5 + \left[\frac{7}{7}\right] 5$$

$$P_{92} = 90.5 + [1]5$$

$$P_{92} = 90.5 + 5$$

$$\boldsymbol{P_{92} = 95.75}$$

References

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Images

Children Thinking Clipart

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Avatars

The avatars used in this module are created using Bitmoji application.

Congratulations!

You are now ready for the next module. Always remember the following:

1. Make sure every answer sheet has your
   - *Name*
   - *Grade and Section*
   - *Title of the Activity or Activity No.*
2. Follow the date of submission of answer sheets as agreed with your teacher.
3. Keep the modules with you and return them at the end of the school year or whenever face-to-face interaction is permitted.